Falsification of Synchronous and Hybrid Systems using Automatic Differentiation

SYNCHRON'19

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Introduction

The bouncing ball

Discrete

```
let node ball y0 = y where
  rec y = y0 → pre (y +. dt *. y_v)

  and y_v = 0. →
    if bounce then
        (-. 0.8 *. pre y_v)
    else
        pre (y_v -. dt *. g)

  and bounce = y < 0.
```

Hybrid

```
let hybrid ball y0 = y where
  rec der y = y_v init y0

  and der y_v = -. g init 0.0
      reset bounce →
        (-. 0.8 *. last y_v)

  and bounce = up(-. y)
```

In this talk: a (very) quick overview of FADBADml[1], a library for Automatic Differentiation in OCaml implemented by François Bidet and myself.

[1] https://fadbadml-dev.github.io/FADBADml/

Summary

- Falsification problem
  - Example
  - State of the art
- Automatic Differentiation : FADBADml
- Falsification with FADBADml
  - A word about specification
  - Example: a simple ODE
  - Synchronous system: the automatic transmission
  - WIP: the heater

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The Falsification problem

Falsification problem Given a System Under Test *SUT* and a specification *spec*, find an input I such that the output O doesn't satisfy $spec(I, O)$

Example: SUT = model of an F-16 aircraft with a Ground Collision Avoidance System (GCAS)

$\hookrightarrow$ **inputs**: initial position and rotation

$\hookrightarrow$ **outputs** at time t: current position, rotation, velocity, angular velocity, ...

$spec$ = the aircraft doesn't crash

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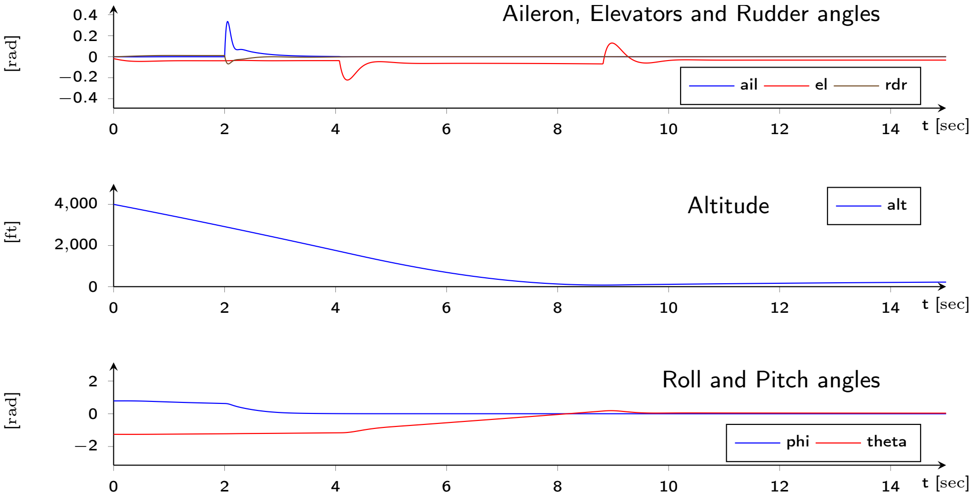
Falsification problem: F-16 aircraft



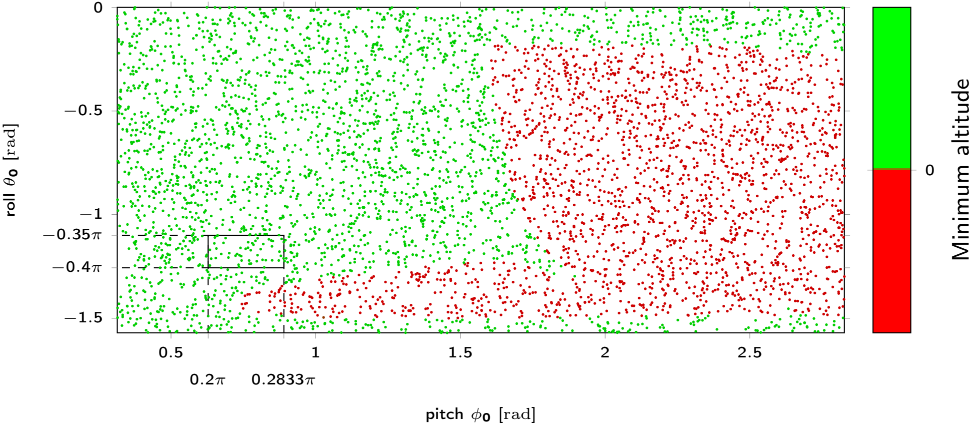
Falsification problem: Uniform Random sampling

Specification : The F16 does not crash

Plot space $\phi_0 \in [0.1\pi, 0.9\pi]$, $\theta_0 \in [-0.5\pi, 0]$

Search space $\phi_0 \in [0.2\pi, 0.2833\pi]$, $\theta_0 \in [-0.4\pi, -0.35\pi]$

Number of points : 5000



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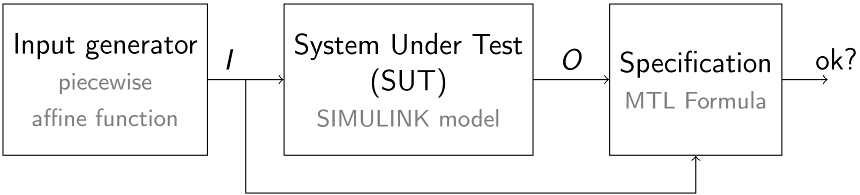
Falsification problem : State of the art

Breach Donzé (2010), S-TaLiro Annpureddy et al. (2011)



Main Idea

instead of asking "Is spec satisfied by SUT ?",
ask "How much is spec satisfied by SUT ?"

$\rightarrow$ compute a robustness (float) instead of a boolean.

The **falsification** problem becomes a **minimization** problem :

$$spec(I, O) \text{ is not satisfied} \Leftrightarrow spec(I, O) < 0$$

Falsification problem : State of the art

other tools: FalStar, falsify

Method	**FalStar**: Two layered falsification with tree search (MCTS Zhang et al. (2018), aLVTS Ernst et al. (2018))
	falsify: Deep Reinforcement Learning Akazaki et al. (2018)

Both methods are **online**: at each step, the tools produce a new input based on the last outputs

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Automatic Differentiation : FADBADml

Porting of FADBAD++ in OCaml by François Bidet and I

FADBAD++ Stauning (1997):

- C++ library for Automatic Differentiation
- written by Ole Stauning as part of his PhD thesis.

FADBADml is available at github and can be installed with opam

Automatic Differentiation : FADBADml

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FADBAD++ Stauning (1997):

- C++ library for Automatic Differentiation
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FADBADml is available at github and can be installed with opam

Example: let `z = x + y`, FADBADml defines a type `F<float>` and overloads the operator + to compute $\frac{dz}{dx}$ and $\frac{dz}{dy}$ at runtime (knowing that $\frac{dx}{dx} = 1$ and $\frac{dy}{dy} = 1$)

Note: FADBAD++ (and FADBADml) implements forward and backward automatic differentiation as well as a library to compute Taylor coefficients of an equation with respect to one variable. However I will only present FAD, the library for forward differentiation.

Automatic Differentiation : FADBADml

How does it work ?

```
(* a,b floats *)
...
let x = make a in
let y = make b in
```

$$x \begin{cases} v = \mathtt{a} \\ \mathit{diff} = [] \end{cases} \qquad y \begin{cases} v = \mathtt{b} \\ \mathit{diff} = [] \end{cases}$$

Automatic Differentiation : FADBADml

How does it work ?

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let x = make a in
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```
diff x 0 2;
diff y 1 2;
```

$$x \begin{cases} v = \texttt{a} \\ \mathit{diff} = \texttt{[1,0]} \end{cases} \qquad y \begin{cases} v = \texttt{b} \\ \mathit{diff} = \texttt{[0,1]} \end{cases}$$

Automatic Differentiation : FADBADml

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```
diff x 0 2;
diff y 1 2;
```

$$x \begin{cases} v = \texttt{a} \\ diff = \texttt{[1,0]} \end{cases} \qquad y \begin{cases} v = \texttt{b} \\ diff = \texttt{[0,1]} \end{cases}$$

```
let z = op(x,y) in
...
(* (d z 0) contains dz/dx
and (d z 1) contains dz/dy *)
```

$$z \begin{cases} v = op_{float}(v_x, v_y) \\ diff = op_{diff}(v_x, v_y, diff_x, diff_y) \end{cases}$$

Example if $op(x, y) = x *_{fad} y$ then
$op_{float}(x, y) = x *_{float} y$ and
$op_{diff}(v_x, v_y, diff_x, diff_y) = v_y * diff_x +_{vec} v_x * diff_y$

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A word about specification

A quantitative semantic of basic constructions

Most simple formulas: usual boolean operators

$$\rho(x > f) = x - f$$

$$\rho(x < f) = f - x$$

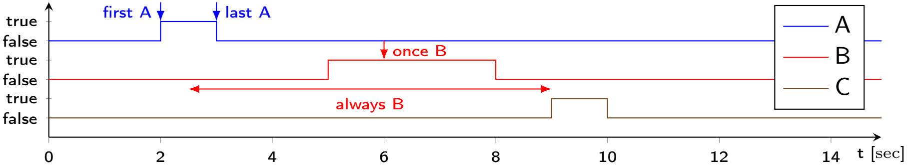
$$\rho(x \vee y) = max(x, y)$$

$$\rho(x \wedge y) = min(x, y)$$

- **Sign**: encodes the boolean value
- **Absolute value**: encodes some kind of score

A word about specification

Temporal constructions



after	first last	A holds,	once always	B holds until C holds

↪ these 4 constructions are enough to express all the properties that I found in several benchmarks Ernst et al. (2019) Dokhanchi et al. (2018) Hoxha et al. (2014)

These macros are synchronous observers Halbwachs et al. (1994)
They express a subset of MITL (Metric Interval Temporal Logic, the logic used by S-TaLiro, Breach, ...)

My contribution on this: quantitative semantics + continuous version

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Falsification with FADBADml

Example: a simple ODE

$$\begin{cases} \dot{x}(t) = x(t) + y(t) + 0.1t \\ \dot{y}(t) = y(t) * cos(2\pi y(t)) + x(t) * sin(2\pi x(t)) + 0.1t \\ x(0) = x_0 \in [-1, 1] \\ y(0) = y_0 \in [-1, 1] \end{cases}$$

Specification: x is always in $[-1.6, -1.4]$

Falsification with FADBADml

Example: a simple ODE

$$
\begin{cases}
\dot{x}(t) = x(t) + y(t) + 0.1t \\
\dot{y}(t) = y(t) * cos(2\pi y(t)) + x(t) * sin(2\pi x(t)) + 0.1t \\
x(0) = x_0 \in [-1, 1] \\
y(0) = y_0 \in [-1, 1]
\end{cases}
$$

Specification: x is always in $[-1.6, -1.4]$

Falsification problem: find $x_0 \in [-1, 1]$ and $y_0 \in [-1, 1]$ such that $\exists t \in [0, t_{max}] \; / \; x(t) \notin [-1.6, -1.4]$

Falsification with FADBADml

Example: a simple ODE

$$\begin{cases} \dot{x}(t) = x(t) + y(t) + 0.1t \\ \dot{y}(t) = y(t) * cos(2\pi y(t)) + x(t) * sin(2\pi x(t)) + 0.1t \\ x(0) = x_0 \in [-1, 1] \\ y(0) = y_0 \in [-1, 1] \end{cases}$$

Specification: x is always in $[-1.6, -1.4]$

Falsification problem: find $x_0 \in [-1, 1]$ and $y_0 \in [-1, 1]$ such that $\exists t \in [0, t_{max}] \ / \ x(t) \notin [-1.6, -1.4]$

(* $(x \geq -1.6) \Leftrightarrow (-1.6 - x \geq 0)$ | $(x \leq -1.4) \Leftrightarrow (x + 1.4 \geq 0)$ *)

Robustness: $\rho(x) = min(-1.6 - x, x + 1.4)$

Falsification with FADBADml

Example: a simple ODE solved with Euler's integration scheme (fixed step)

System

$$\begin{cases} \dot{x}(t) = x(t) + y(t) + 0.1t \\ \dot{y}(t) = y(t) * cos(2\pi y(t)) + x(t) * sin(2\pi x(t)) + 0.1t \\ x(0) = x_0 \\ y(0) = y_0 \end{cases}$$

Execution (`dt` is a fixed parameter)

```
rec t = 0 → (pre t) + dt
and x = (pre x) + dt * ((pre x) + (pre y) + 0.1 * t)
and y = (pre y) + dt * ((pre y) * cos(2*pi*(pre y)) +
                        (pre x) * sin(2*pi*(pre x)) + 0.1 * t)
```

With FADBADml, we can compute dx/dx_0 and dx/dy_0 after any number of steps

Falsification with FADBADml

Example: a simple ODE with Euler and Gradient Descent Reddi et al. (2019)

Specification

$$x \in [-1.6, -1.4] \wedge$$
$$y \in [-1.1, -0.9]$$

Parameters

$$\text{input space} = [-1, 1] \; x \; [-1, 1]$$
$$\texttt{dt} = 0.1$$
$$\texttt{n\_steps} = 100$$

Falsification with FADBADml

Example: a simple ODE with Euler and Gradient Descent Reddi et al. (2019)

Specification

$$x \in [-1.6, -1.4] \wedge$$
$$y \in [-1.1, -0.9]$$

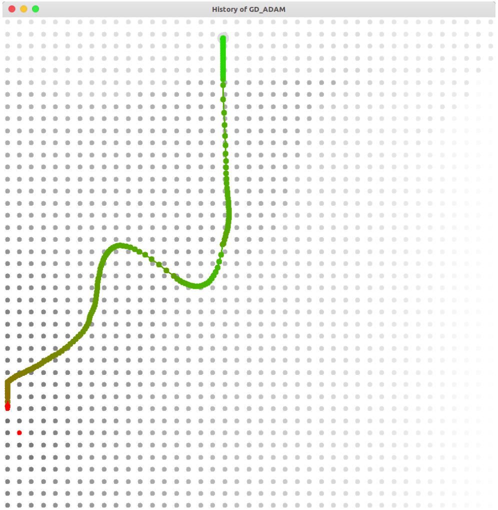
Parameters

$$\text{input space} = [-1, 1] \ x \ [-1, 1]$$
$$\texttt{dt} = 0.1$$
$$\texttt{n\_steps} = 100$$

Falsified after 172 tries:

- best rob = -0.0115205
- sample =

$$\begin{cases} x_0 = -1. \\ y_0 = -0.589587520984 \end{cases}$$



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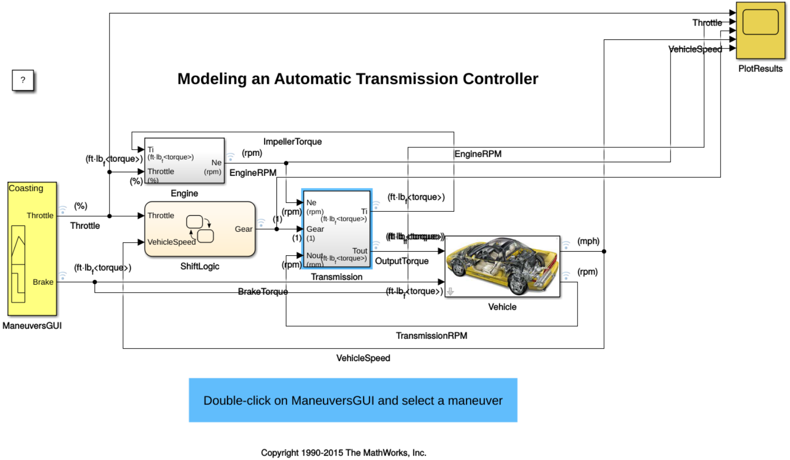
Falsification with FADBADml

Synchronous system: automatic transmission (ARCH Comp. 2014) Hoxha et al. (2014)Hoxha et al. (2015)

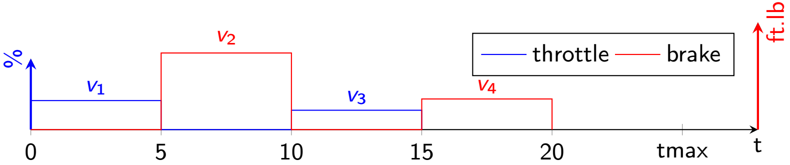


Input: `throttle`, `brake` **Output**: `gear`, `speed`, `rpm`

Falsification with FADBADml

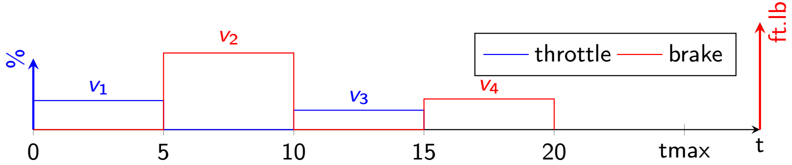
Synchronous system: the automatic transmission, offline falsification

Input shape (dim. 4)



Falsification with FADBADml

Synchronous system: the automatic transmission, offline falsification

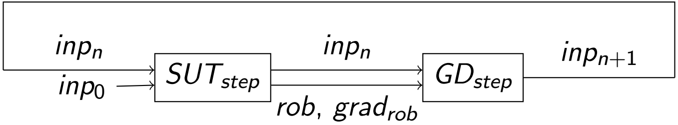
Input shape (dim. 4)	
Specification	The engine and the vehicle speed never reach $\bar{w}$ and $\bar{v}$ resp.
Robustness	$\rho(w, v) = min(\bar{w} - w, \bar{v} - v)$

This is the same as before: instead of picking 2 values at the beginning, we pick 4 of them and construct the input with them

Falsification with FADBADml

Synchronous system: the automatic transmission, online falsification

Online falsification :



DEMO

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Falsification with FADBADml

WIP: the heater (Nicolas Halbwachs, Collège de France, 2010)

```
let node euler(h)(x0, xprime) = x
  where rec x = x0 → pre (x +. h *. xprime)

let node heater(c, α, β, temp_ext, temp0, u) = temp where
  rec der_temp =
    if u then (c - temp) *. α
         else (temp_ext - temp) *. β
  and temp = euler(0.2)(temp0, der_temp)

 let node relay(low, high, v) = u where
   rec u = if v < low then true
          else if v > high then false
          else false → pre u

let node system(reference) = (u, temp) where
  rec u = relay(reference -. 1., reference +. 1., temp)
  and temp = heater(50.0, 0.1, 0.05, 0.0, 15.0, u)
```

Falsification with FADBADml

WIP: the heater

$$\dot{temp} = \begin{cases} \alpha * (c - temp) & \text{if } u \\ \beta * (temp_{ext} - temp) & \text{if } \neg u \end{cases}$$

integrated using euler

$$temp_{n+1} = temp_n + \delta_n * \begin{cases} \alpha * (c - temp_n) & \text{if } u_n \\ \beta * (temp_{ext} - temp_n) & \text{if } \neg u_n \end{cases}$$

$$u_{n+1} = \begin{cases} true & \text{if } temp_n < ref - low \\ false & \text{if } temp_n > ref - high \\ u_n & \text{else} \end{cases}$$

Falsification with FADBADml

WIP: the heater

$$temp_{n+1} = temp_n + \delta_n * \begin{cases} \alpha * (c - temp_n) & \text{if } u_n \\ \beta * (temp_{ext} - temp_n) & \text{if } \neg u_n \end{cases}$$

and $temp_0$ is a constant

$$u_{n+1} = \begin{cases} true & \text{if } temp_n < ref - low \\ false & \text{if } temp_n > ref - high \\ u_n & \text{else} \end{cases}$$

and $u_0 = false$

Falsification with FADBADml

WIP: the heater

$$temp_{n+1} = temp_n + \delta_n * \begin{cases} \alpha * (c - temp_n) & \text{if } u_n \\ \beta * (temp_{ext} - temp_n) & \text{if } \neg u_n \end{cases}$$

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and $u_0 = false$

We take the partial derivative of *temp* w.r.t. *ref*

$$\frac{d}{dref} temp_{n+1} = \frac{d}{dref} temp_n + \delta_n * \begin{cases} -\alpha * \frac{d}{dref} temp_n & \text{if } u_n \\ -\beta * \frac{d}{dref} temp_n & \text{if } \neg u_n \end{cases}$$

Falsification with FADBADml

WIP: the heater

$$temp_{n+1} = temp_n + \delta_n * \begin{cases} \alpha * (c - temp_n) & \text{if } u_n \\ \beta * (temp_{ext} - temp_n) & \text{if } \neg u_n \end{cases}$$

and $temp_0$ is a constant

$$u_{n+1} = \begin{cases} true & \text{if } temp_n < ref - low \\ false & \text{if } temp_n > ref - high \\ u_n & \text{else} \end{cases}$$

and $u_0 = false$

We take the partial derivative of $temp$ w.r.t. ref

$$\frac{d}{dref} temp_{n+1} = \frac{d}{dref} temp_n + \delta_n * \begin{cases} -\alpha * \frac{d}{dref} temp_n & \text{if } u_n \\ -\beta * \frac{d}{dref} temp_n & \text{if } \neg u_n \end{cases}$$

Let's apply with $n = 0$

$$\frac{d}{dref} temp_1 = 0 + \delta_0 * \begin{cases} -\alpha * 0 & \text{if } u_0 \\ -\beta * 0 & \text{if } \neg u_0 \end{cases}$$

Falsification with FADBADml

WIP: the heater

What happens if we trigger mode transitions ?

First: quantitative semantics over boolean formulas :

$$u_{n+1} = \begin{cases} (ref - low) - temp_n & \text{if } temp_n < ref - low \\ temp_n - (ref - high) & \text{if } temp_n > ref - high \\ u_n & \text{else} \end{cases}$$

Falsification with FADBADml

WIP: the heater

What happens if we trigger mode transitions ?

First: quantitative semantics over boolean formulas :

$$u_{n+1} = \begin{cases} (ref - low) - temp_n & \text{if } temp_n < ref - low \\ temp_n - (ref - high) & \text{if } temp_n > ref - high \\ u_n & \text{else} \end{cases}$$

Then: derivatives w.r.t. the input

$$\frac{d}{dref} u_{n+1} = \begin{cases} 1 - \frac{d}{dref} temp_n & \text{if } temp_n < ref - low \\ \frac{d}{dref} temp_n - 1 & \text{if } temp_n > ref - high \\ \frac{d}{dref} u_n & \text{else} \end{cases} = \begin{cases} 1 & \text{if } temp_n < ref - low \\ -1 & \text{if } temp_n > ref - high \\ \frac{d}{dref} u_n & \text{else} \end{cases}$$

Falsification with FADBADml

WIP: the heater

```
let node relay(low, high, v) = u where
  automaton
  | HIGH →
    do u = false
    until (v < low) then LOW             (* zc: "trigger_low", rob: low - v *)
    else (v < high) then MID(false)      (* zc: "trigger_mid", rob: high - v *)
  | MID(out) →
    do u = out
    until (v > high) then HIGH           (* zc: "trigger_high", rob: v - high *)
    else (v < low) then LOW              (* zc: "trigger_low", rob: low - v *)
  | LOW →
    do u = true
    until (v > high) then HIGH           (* zc: "trigger_high", rob: v - high *)
    else (v > low) then MID(true)        (* zc: "trigger_mid", rob: v - low *)
```

Falsification with FADBADml

WIP: the heater

Specification: once the temperature is in $[ref - low, ref + high]$, it always stays in $[ref - 2 * low, ref + 2 * high]$

DEMO

References I

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