

Reactive Probabilistic Programming

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Probabilistic Programming

Probabilistic Programming

Programming and reasoning with uncertainty

- Program functions with uncertainty: sample from distributions
- Condition on observed data: inputs of the model

Probabilistic Programming Languages

- Bugs, Stan, Church, Blog, Anglican, Venture, Figaro, WebPL, Pyro, Edward, ...

Probabilistic constructs:

- $x = \text{sample}(d)$: introduce a random variable x of distribution d
- $\text{observe}(d, y)$: measure the likelihood of an observation y w.r.t d
- $\text{infer } m \text{ obs}$: compute output distribution of a model m given obs

Inference: compute probability distribution defined by a model given observations or data (similar to learning in machine learning)

ProbZelus: Design Choices

Zelus extended with probabilistic constructs

Inference in the loop

- Interaction between deterministic processes and probabilistic models
- Models receives input from the environment
- Deterministic processes can access intermediate results
- Feedback between inferred distribution and deterministic processes

Streaming inference

- Inference runs in parallel with deterministic processes (non-terminating)
- Should run with bounded resources
- Multiples inference algorithms with different trade-off cost/accuracy
- Sequential Monte-Carlo (SMC) vs. streaming delayed sampling

Bayesian Inference (in-the-loop)

Learn parameters from data

- Latent parameters at instant t θ_t
- Observed data x_1, \dots, x_t

Compute the distribution $p(\theta_t | x_1, \dots, x_t)$ at each time step

$$p(\theta_t | x_1, \dots, x_t) = \frac{p(\theta_t)p(x_1, \dots, x_t | \theta_t)}{p(x_1, \dots, x_t)} \quad (\text{Bayes' theorem})$$

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$$\propto p(\theta_t)p(x_1, \dots, x_t | \theta_t) \quad (\text{Data are constants})$$

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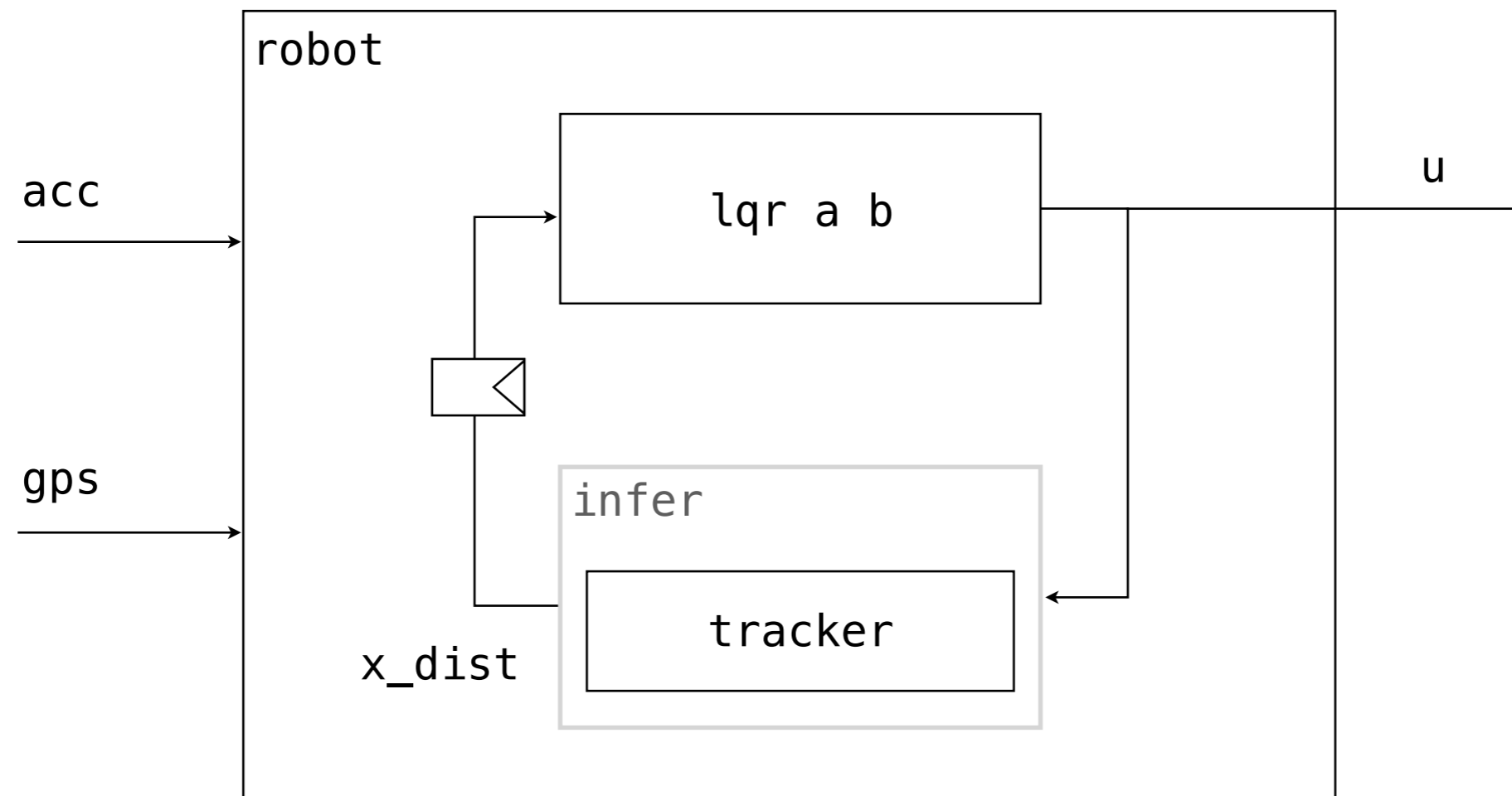
prior: **sample**

likelihood: **observe**

Example

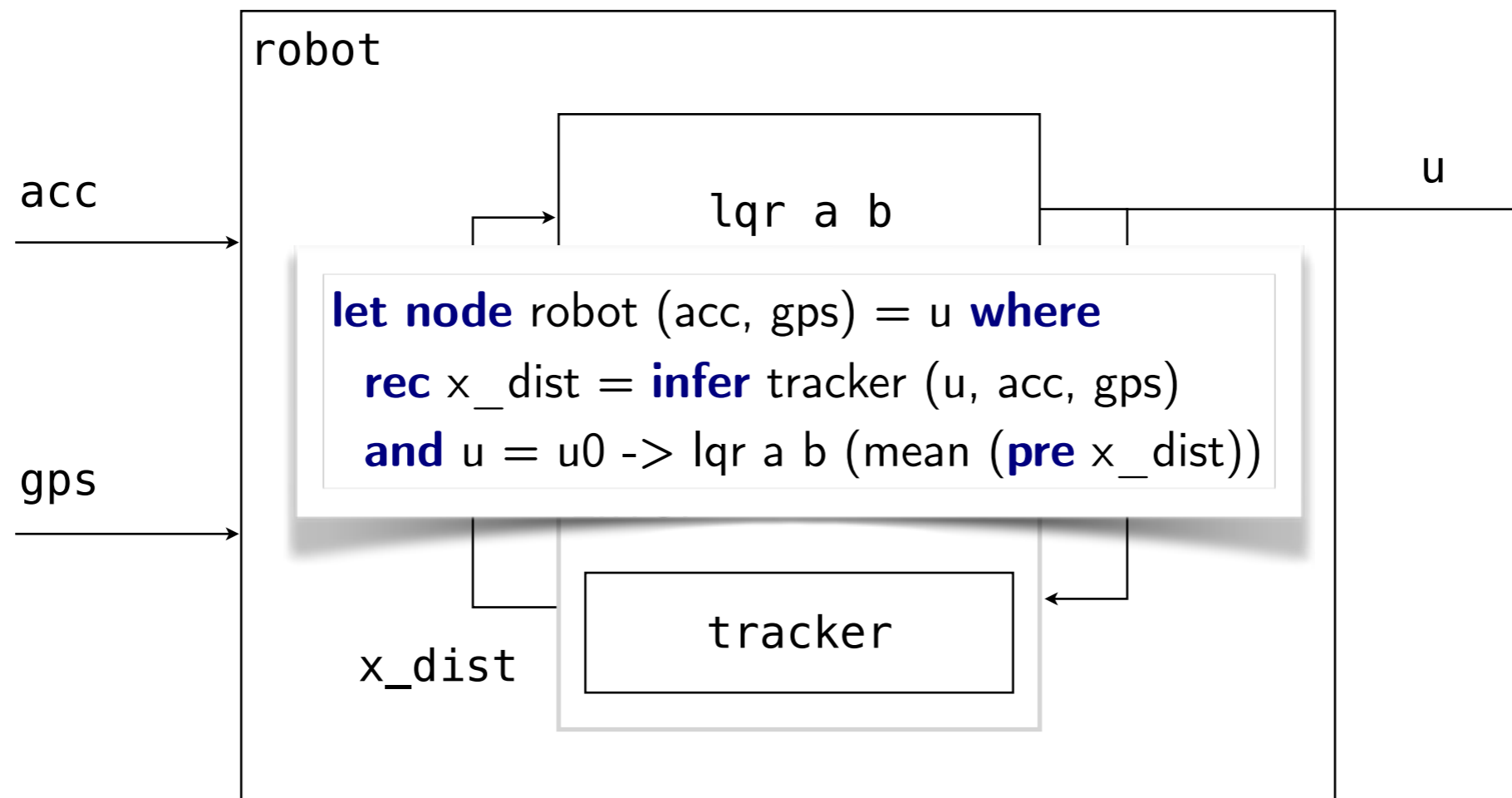
Robot Controller

- Input: noisy acceleration **acc** (at each step), noisy position **gps** (sporadic)
- Output: command **u** to drive the robot to a given target
- State: $x_t = (\text{position, velocity, acceleration})$
- Motion model: $x_{t+1} = A.x_t + B.u_t$ (A, B are constant matrices)

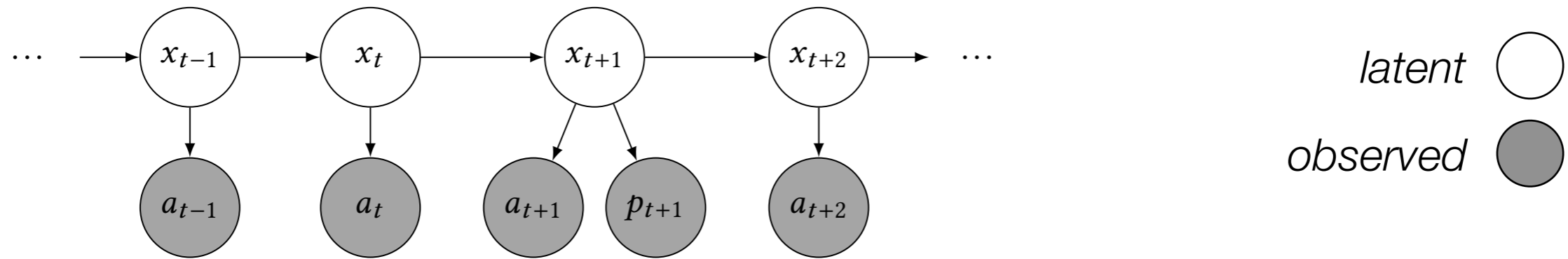


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Robot Controller



let proba kalman (u, acc, gps) = x **where**

rec mu = x0 -> (a *@ pre x) +@ (b *@ u) (* x_{t+1} = A.x_t + B.u_t *)

and x = **sample** (mv_gaussian (mu, noise))

and () = **observe** (gaussian (vec_get x 2, 1.0), acc)

and present gps (pos) ->

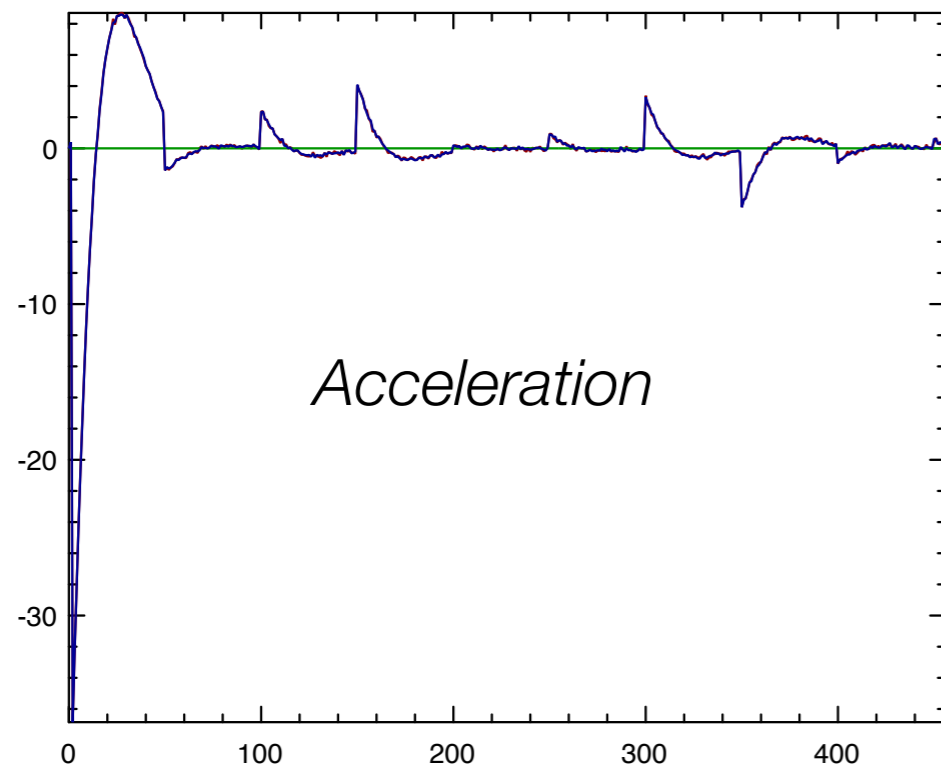
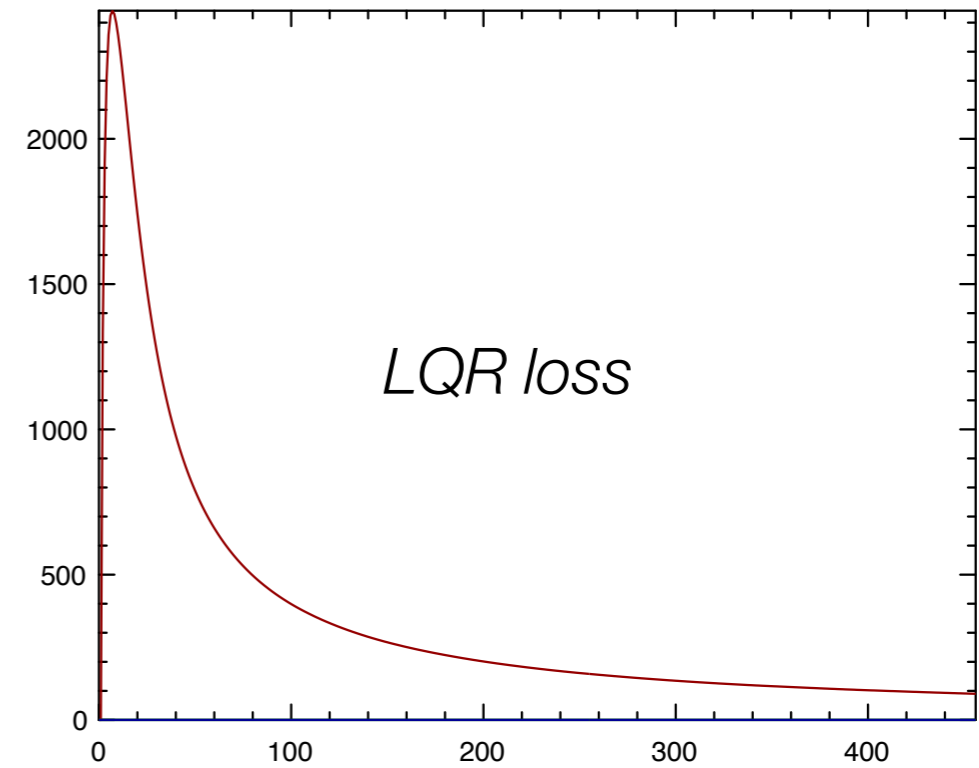
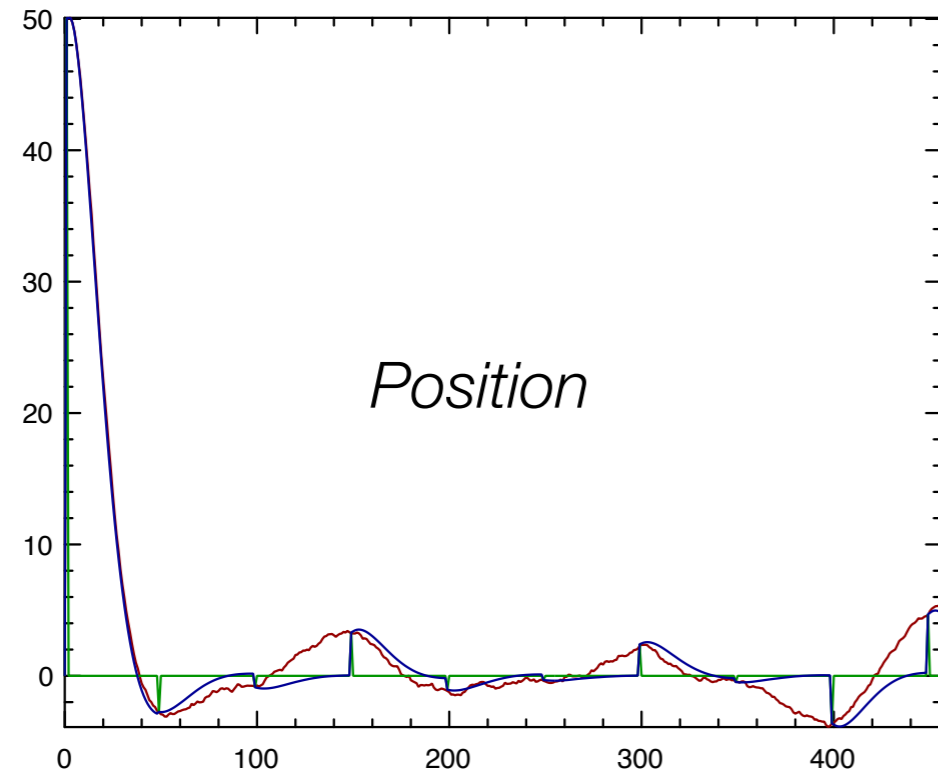
do () = **observe** (gaussian (vec_get x 0, 0.01), pos) **done**

let node robot (acc, gps) = u **where**

rec x_dist = **infer** 100 tracker (u, acc, gps)

and u = u0 -> lqr a b (mean (**pre** x_dist))

Robot Controller



- *exact value*
- *estimated value*
- *gps readings*

Language

Syntax

$d ::= \text{let node } f \ x = e \mid \text{let proba } f \ x = e \mid d \ d$
 $e ::= c \mid x \mid (e, e) \mid op(e) \mid f(e) \mid \text{last } x \mid e \ \text{where rec } E$
 $\mid \text{present } e \rightarrow e \ \text{else } e \mid \text{reset } e \ \text{every } e$
 $\mid \text{sample}(e) \mid \text{observe}(e, e) \mid \text{infer}(e)$
 $E ::= x = e \mid \text{init } x = c \mid E \ \text{and } E$

- Other constructs can be expressed in this kernel.
- Probabilistic models are nodes (**proba**)
- Local equations in e **where rec** E are scheduled
x **where**
 rec init $x1 = c1$
 and init $x2 = c2$
 and $x1 = e1$
 and $x2 = e2$

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\times **where**

```
rec init x1 = c1  
and init x2 = c2  
and x1 = e1  
and x2 = e2
```

```
x = 0 -> pre x + 1
```

\times **where**

```
rec init fst = true  
and init x = 0  
and fst = false  
and x = if last fst then 0 else last x + 1
```


Typing: D vs. P

$$\frac{G \vdash^D e : T \text{ dist}}{G \vdash^P \text{sample}(e) : T}$$

$$\frac{G \vdash^D e : T}{G \vdash^P e : T}$$

$$\frac{G \vdash^D e_1 : T \text{ dist} \quad G \vdash^D e_2 : T}{G \vdash^P \text{observe}(e_1, e_2) : \text{unit}}$$

$$\frac{G \vdash^P e : T}{G \vdash^D \text{infer}(e) : T \text{ dist}}$$

- Add a kind D (deterministic) or P (probabilistic)
- **sample** and **observe** can only be used in a probabilistic context
- Deterministic expression can be lifted to probabilistic ones
- Transition realized by **infer**
- Add a datatype for distributions $T \text{ dist}$

Co-iteration Semantics

Deterministic Stream: Initial state, transition function

$$\mathit{CoStream}(T, S) = S \times (S \rightarrow T \times S)$$

$$\mathit{CoNode}(T, T', S) = S \times (S \rightarrow T \rightarrow T' \times S).$$

$$\llbracket e \rrbracket_{\gamma} : \mathit{CoStream}(T, S) = \llbracket e \rrbracket_{\gamma}^i, \llbracket e \rrbracket_{\gamma}^s$$

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Probabilistic Stream: transition function returns a *measure* over pairs (result, state)

$$\mathit{CoPStream}(T, S) = S \times (S \rightarrow (\Sigma_{T \times S} \rightarrow [0, \infty]))$$

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Normalize the measure to obtain a distribution

$$\begin{aligned} \mu &: \Sigma_D \rightarrow [0, \infty] \\ d : T \text{ dist} &= \frac{\mu}{\int_D \mu(dx)} \end{aligned}$$

Co-iteration Semantics: D

$$\begin{aligned} \llbracket x \rrbracket_{\gamma}^i &= () \\ \llbracket x \rrbracket_{\gamma}^s &= \lambda s. (\gamma(x), s) \end{aligned}$$

$$\begin{aligned} \llbracket \text{present } e \rightarrow e_1 \text{ else } e_2 \rrbracket_{\gamma}^i &= (\llbracket e \rrbracket_{\gamma}^i, \llbracket e_1 \rrbracket_{\gamma}^i, \llbracket e_2 \rrbracket_{\gamma}^i) \\ \llbracket \text{present } e \rightarrow e_1 \text{ else } e_2 \rrbracket_{\gamma}^s &= \lambda(s, s_1, s_2). \text{ let } v, s' = \llbracket e \rrbracket_{\gamma}^s(s) \text{ in} \\ &\quad \text{if } v \text{ then let } v_1, s'_1 = \llbracket e_1 \rrbracket_{\gamma}^s(s_1) \text{ in } (v_1, (s', s'_1, s_2)) \\ &\quad \text{else let } v_2, s'_2 = \llbracket e_2 \rrbracket_{\gamma}^s(s_2) \text{ in } (v_2, (s', s_1, s'_2)) \end{aligned}$$

$$\left[\begin{array}{l} e \text{ where} \\ \text{rec init } x_1 = c_1 \text{ and init } x_2 = c_2 \\ \text{and } x_1 = e_1 \text{ and } x_2 = e_2 \end{array} \right]_{\gamma}^i = \left(\begin{array}{c} (c_1, c_2), \\ (\llbracket e_1 \rrbracket_{\gamma}^i, \llbracket e_2 \rrbracket_{\gamma}^i), \\ \llbracket e \rrbracket_{\gamma}^i \end{array} \right)$$

$$\lambda((m_1, m_2), (s_1, s_2), s).$$

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Co-iteration Semantics: P

$$\begin{aligned}\{\{e\}_\gamma^i &= \llbracket e \rrbracket_\gamma^i && \text{if } \text{kindOf}(e) = D \\ \{\{e\}_\gamma^s &= \lambda s. \lambda U. \delta_{\llbracket e \rrbracket_\gamma^s(s)}(U) && \text{if } \text{kindOf}(e) = D\end{aligned}$$

$$\begin{aligned}\{\{\text{sample}(e)\}_\gamma^i &= \llbracket e \rrbracket_\gamma^i \\ \{\{\text{sample}(e)\}_\gamma^s &= \lambda s. \lambda U. \text{let } \mu, s' = \llbracket e \rrbracket_\gamma^s(s) \text{ in } \int_T \mu(dv) \delta_{v,s'}(U)\end{aligned}$$

$$\begin{aligned}\{\{\text{observe}(e_1, e_2)\}_\gamma^i &= (\llbracket e_1 \rrbracket_\gamma^i, \llbracket e_2 \rrbracket_\gamma^i) \\ \{\{\text{observe}(e_1, e_2)\}_\gamma^s &= \lambda(s_1, s_2). \lambda U. \\ &\quad \text{let } \mu, s'_1 = \llbracket e_1 \rrbracket_\gamma^s(s_1) \text{ in} \\ &\quad \text{let } \nu, s'_2 = \llbracket e_2 \rrbracket_\gamma^s(s_2) \text{ in } \mu_{\text{pdf}}(\nu) * \delta_{(),(s'_1, s'_2)}(U)\end{aligned}$$

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Co-iteration Semantics: infer

$$\begin{aligned} \llbracket \text{infer}(e) \rrbracket_Y^i &= \lambda U. \delta_{\llbracket e \rrbracket_Y^i}(U) \\ \llbracket \text{infer}(e) \rrbracket_Y^s &= \lambda \sigma. \text{let } \mu = \lambda U. \frac{\int_S \sigma(ds) \llbracket e \rrbracket_Y^s(s)(U)}{\int_S \sigma(ds) \llbracket e \rrbracket_Y^s(s)(\top)} \text{ in } (\pi_{1*}(\mu), \pi_{2*}(\mu)) \end{aligned}$$

- The state of **infer** is a distribution
- At each step **infer** compute a distribution of results, and a distribution of states
- Free variables in e capture input from deterministic processes
- The distribution of results can be used by other deterministic processes
- The distribution of state is used for the next step

Inference-in-the-loop

Inference

Particle Filtering

Launch N particles. At each step:

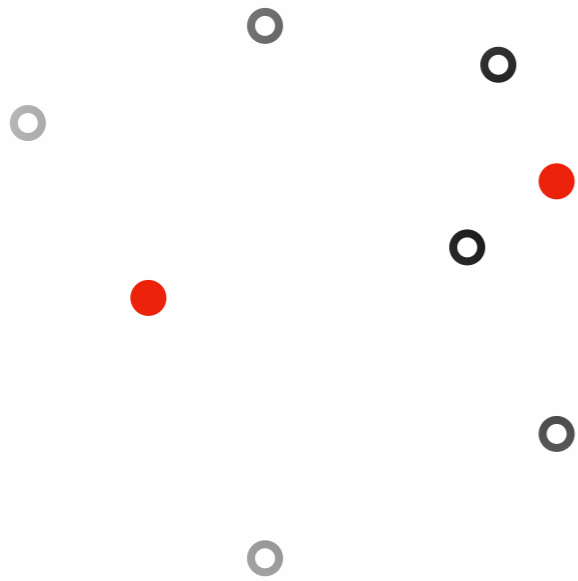
- Each particle generate pairs (result, score) with an importance score
- Normalize the pairs based on the score
- Re-sample a new set of particles from this distribution



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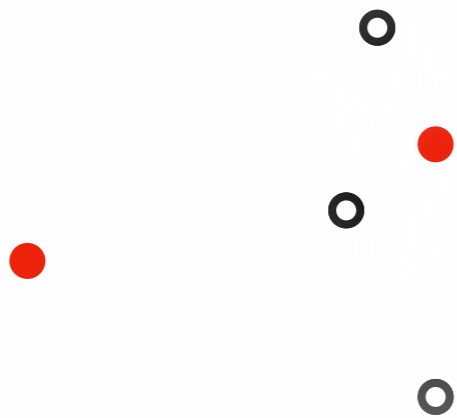
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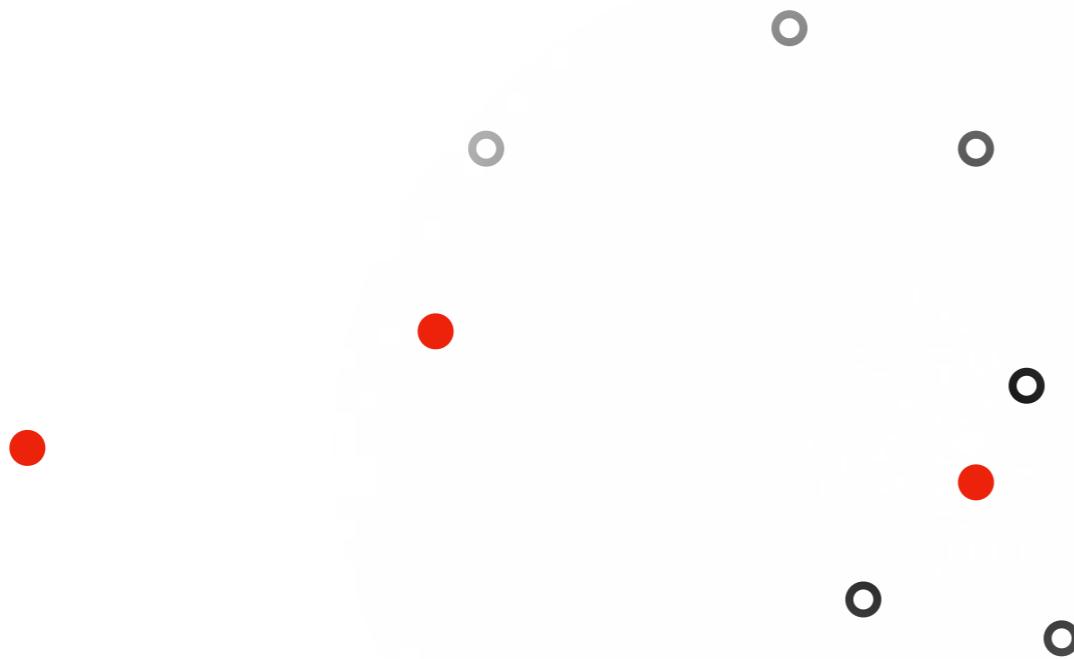
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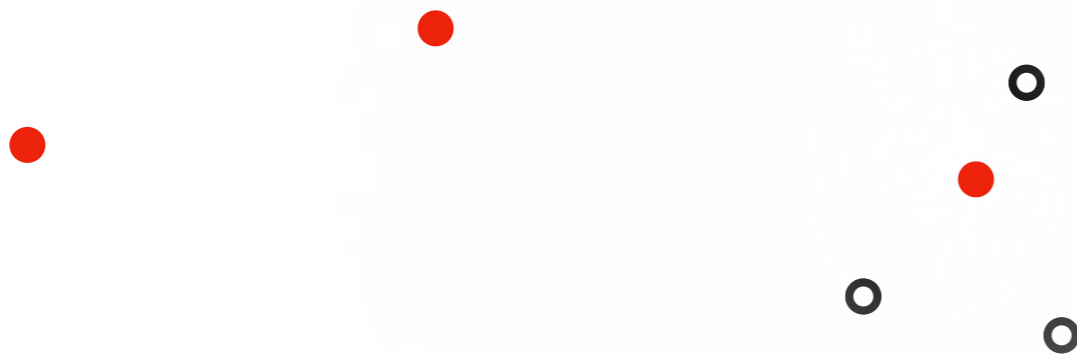
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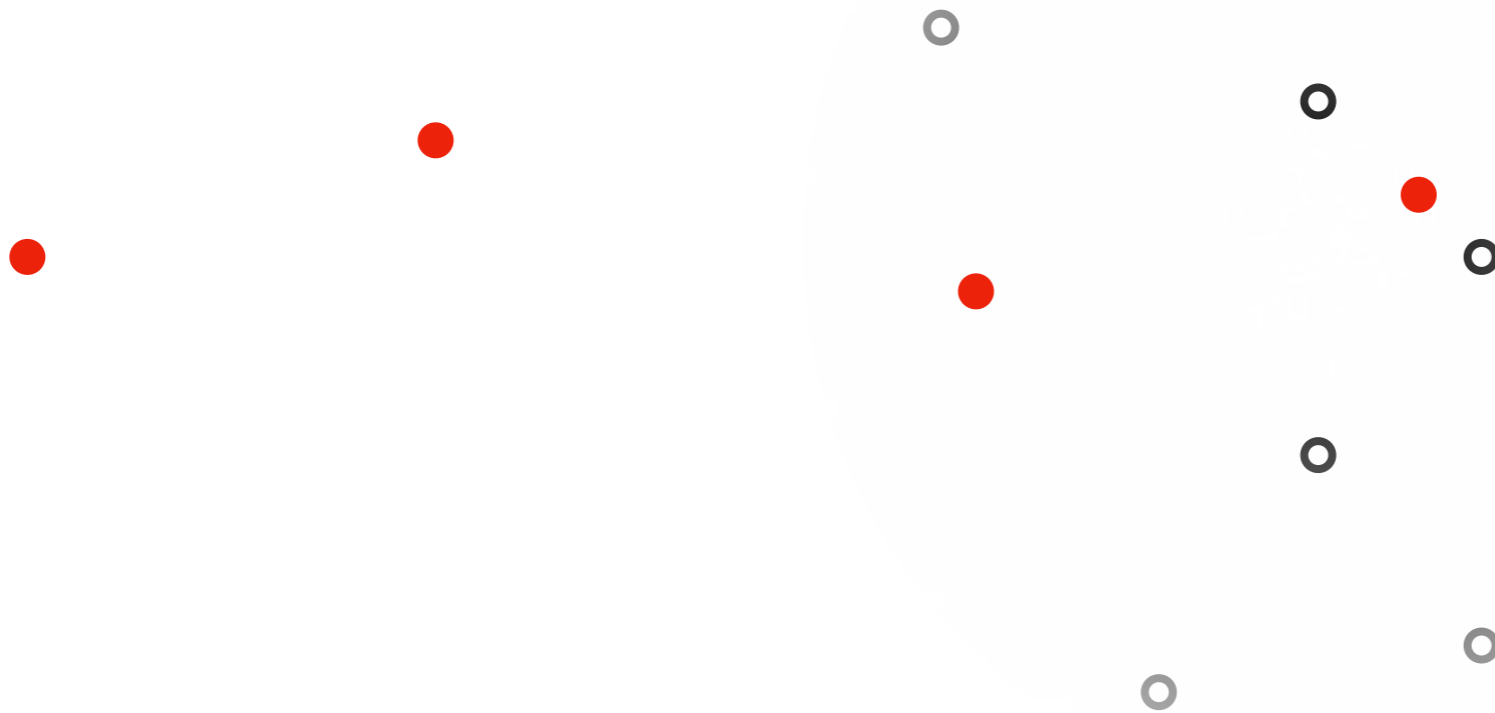
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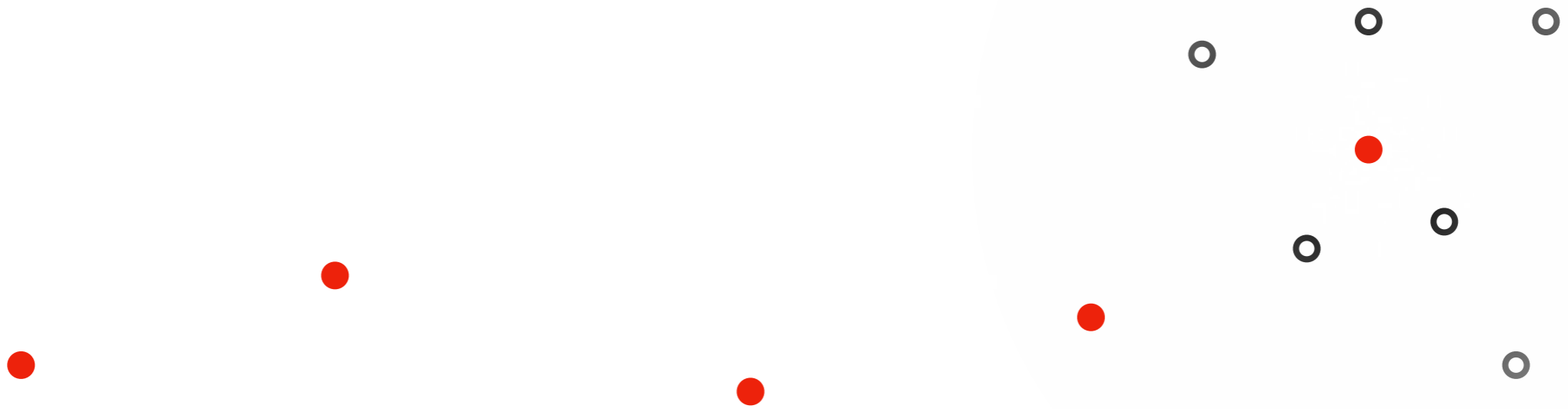
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Delayed Sampling

Particle filter + symbolic computations

- Exploit relations between random variables to maintain a *Bayesian network*
- Observation can be incorporated by analytically *conditioning* the network
- Exact solution if possible, default to particle filtering otherwise



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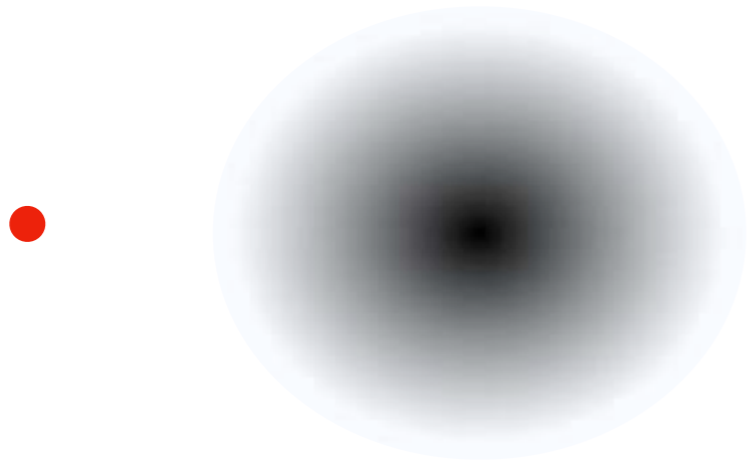
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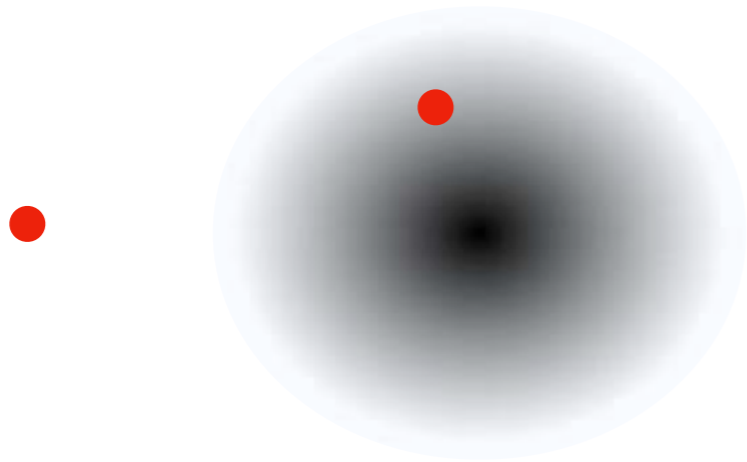
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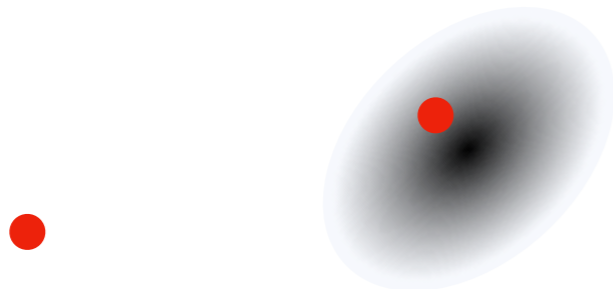
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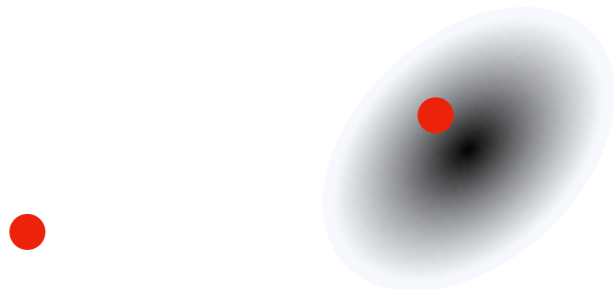
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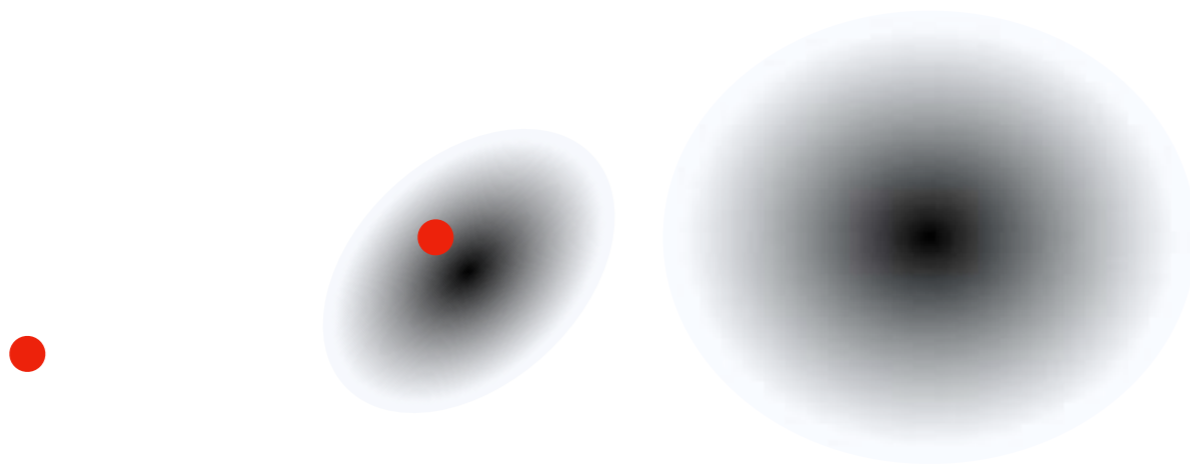
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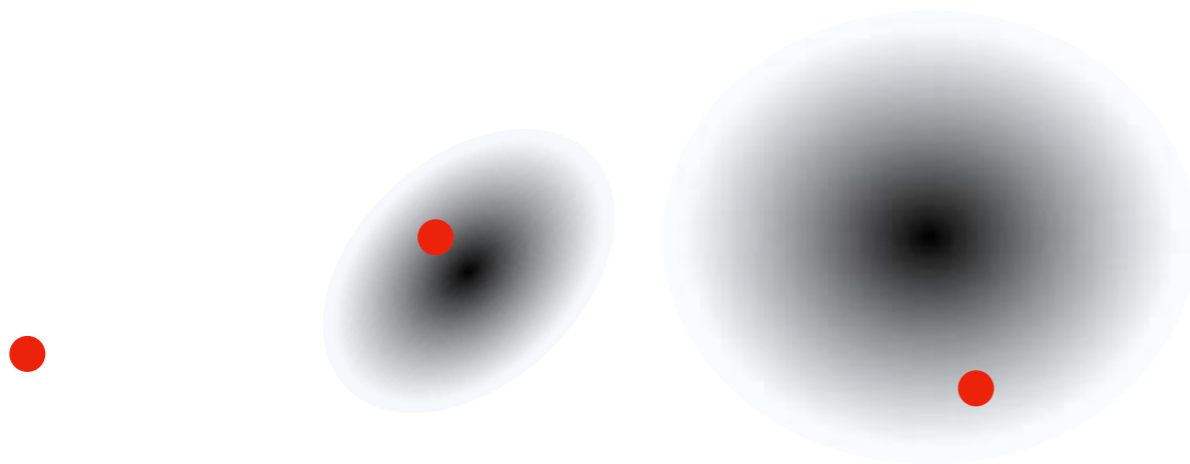
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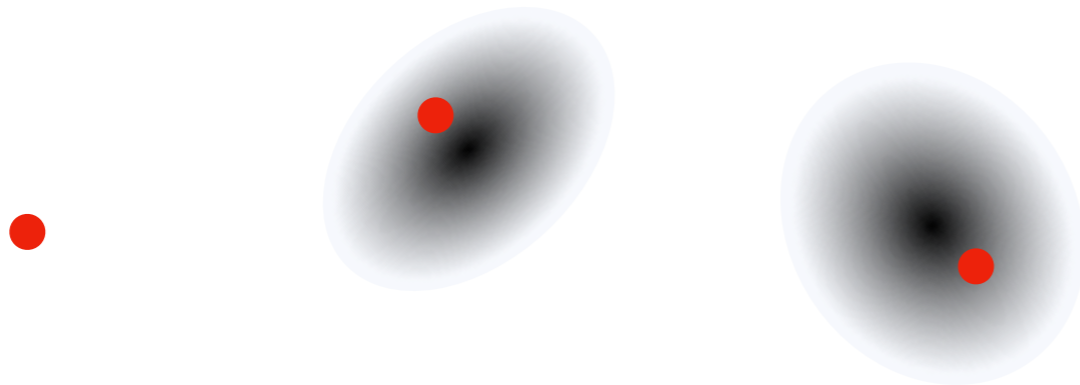
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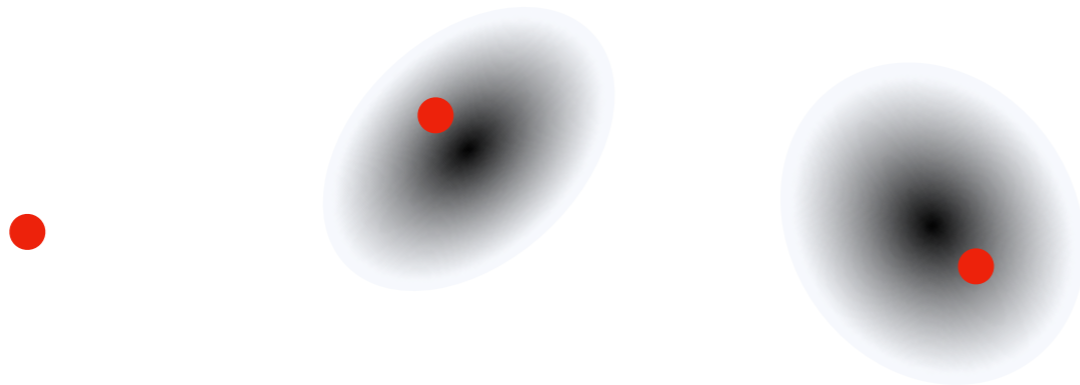
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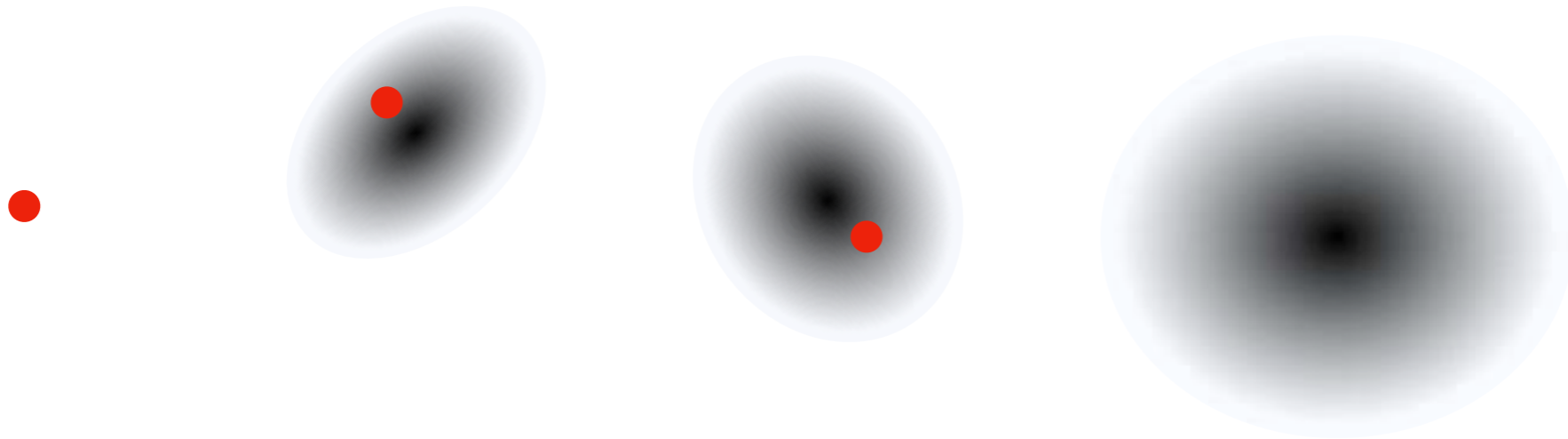
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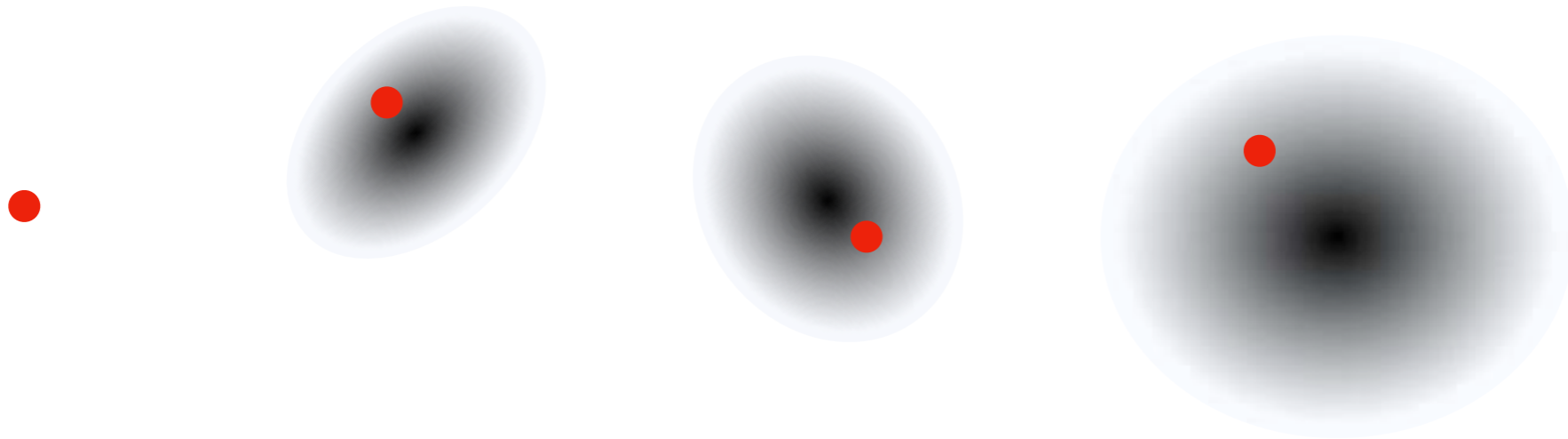
- Exploit relations between random variables to maintain a *Bayesian network*
- Observation can be incorporated by analytically *conditioning* the network
- Exact solution if possible, default to particle filtering otherwise



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Particle filter + symbolic computations

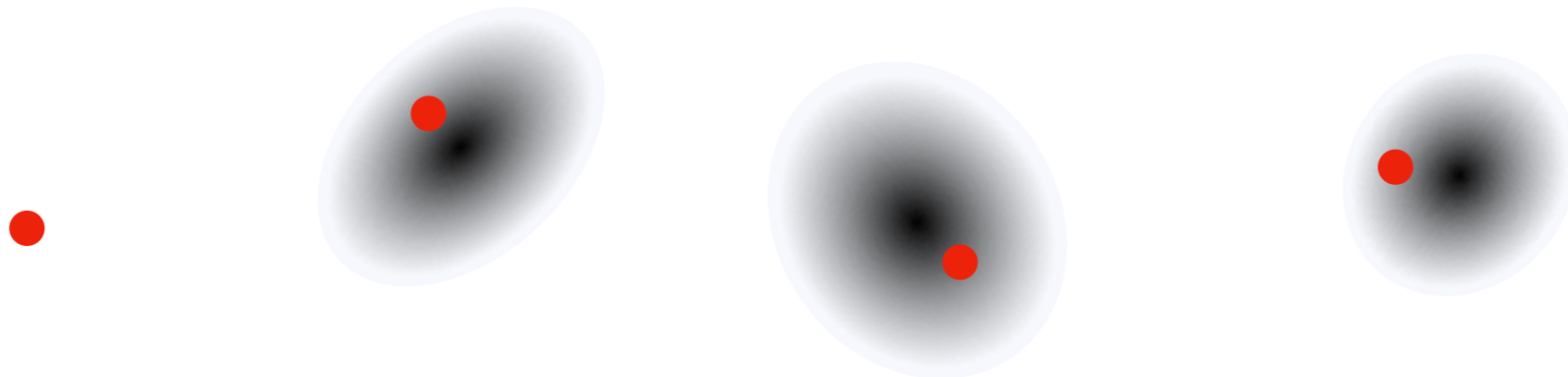
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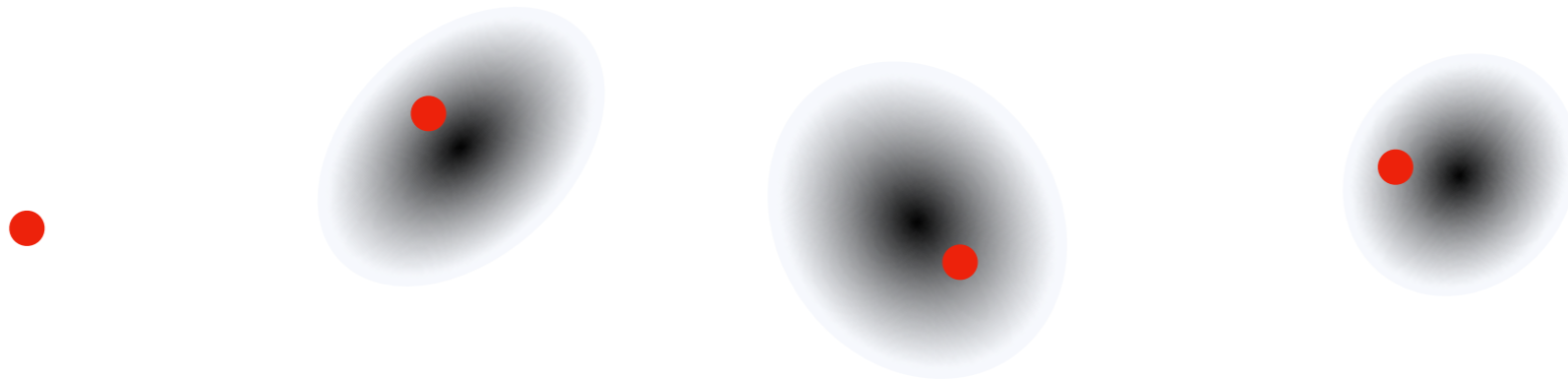
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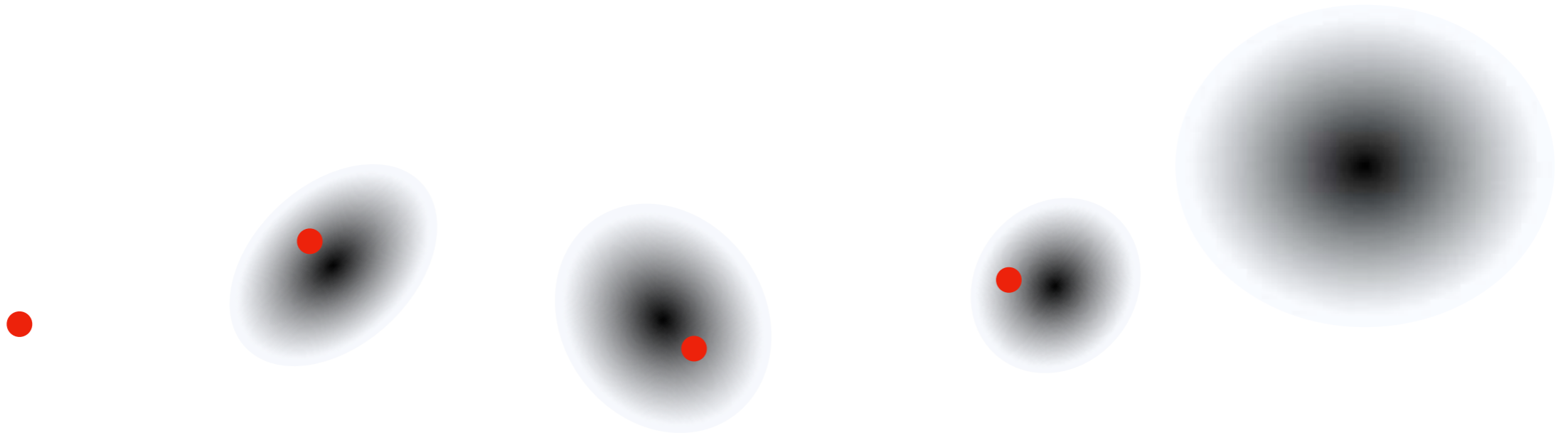
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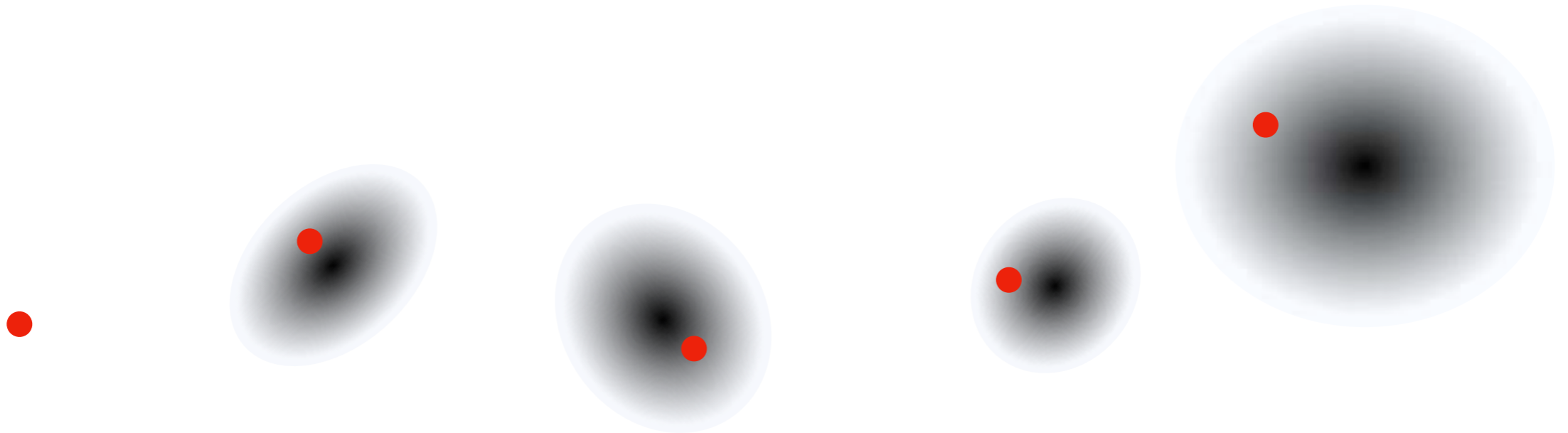
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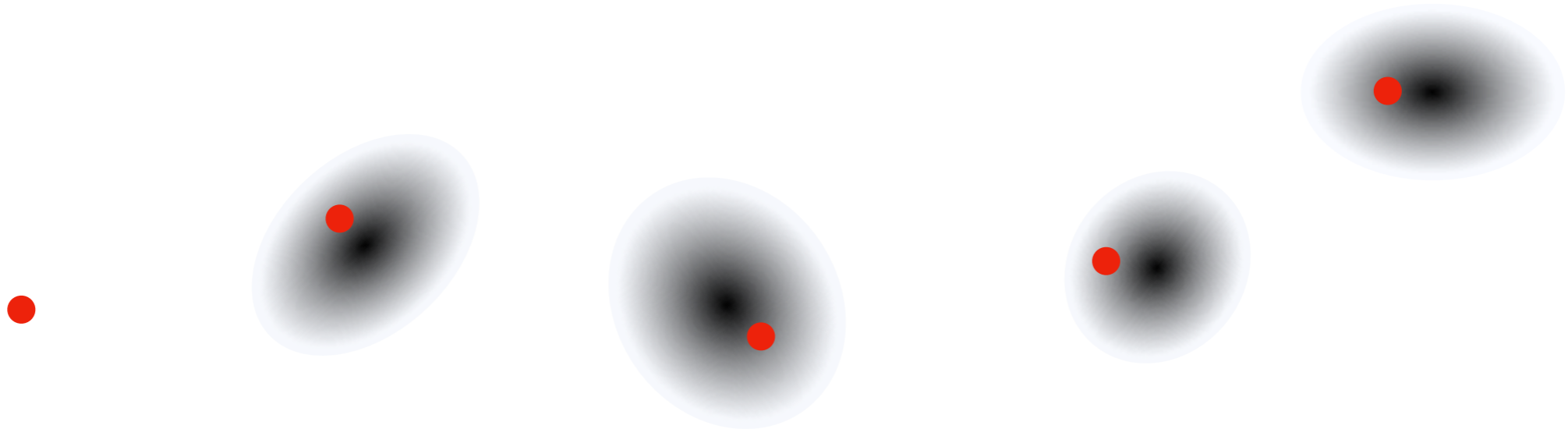
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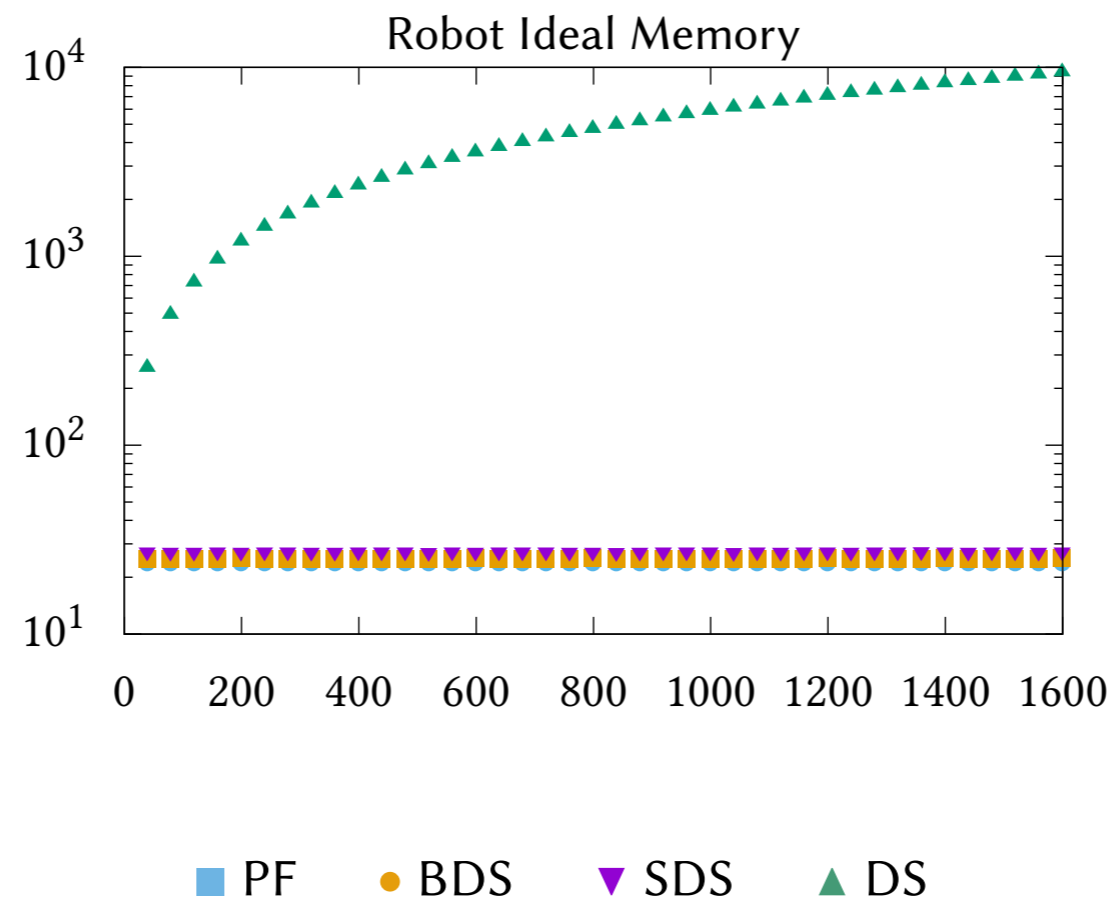
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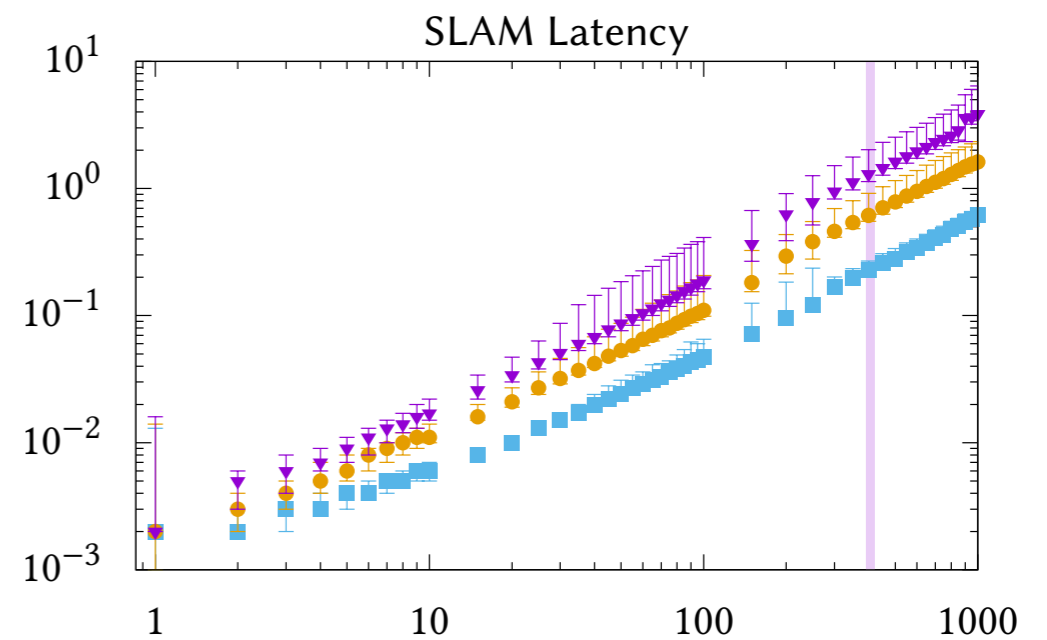
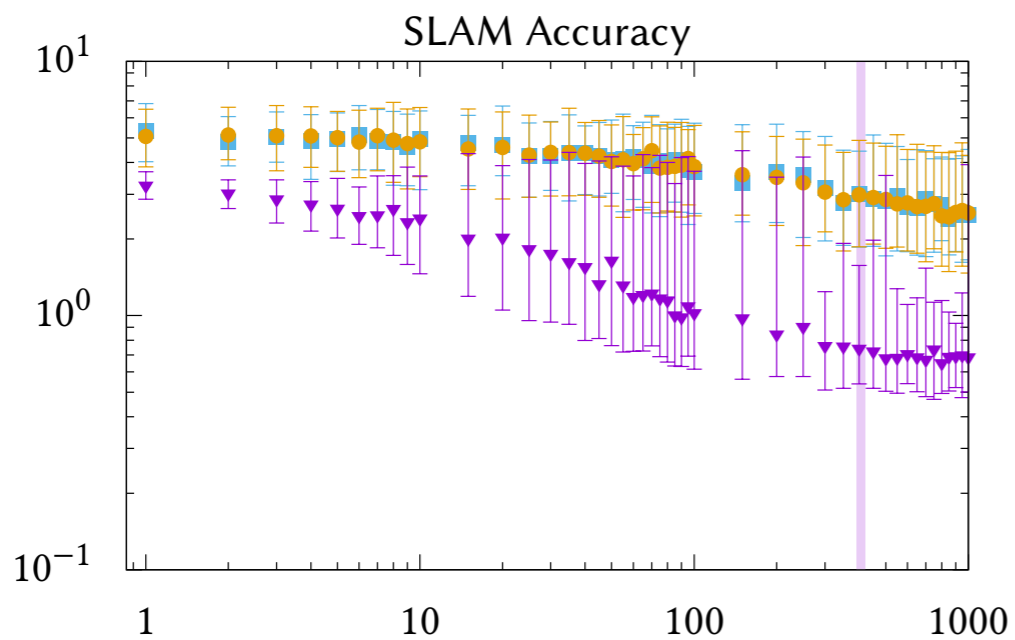
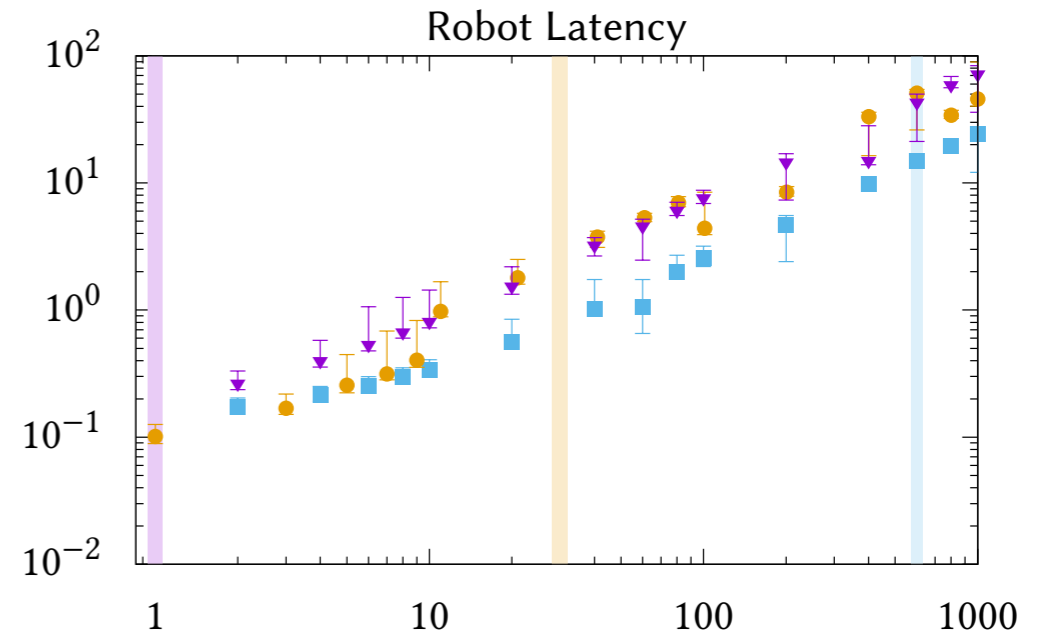
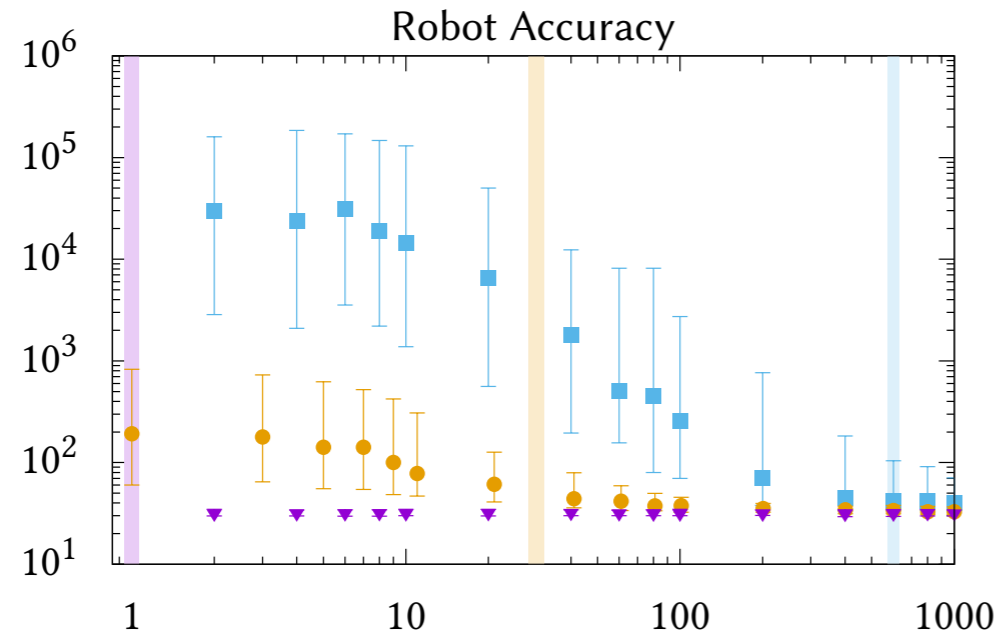
Streaming Delayed Sampling

- Problem: the size of the network is linear in the number of samples
- Novel implementations (SDS, and BDS) run in bounded memory



Streaming Delayed Sampling

■ PF ● BDS ▼ SDS

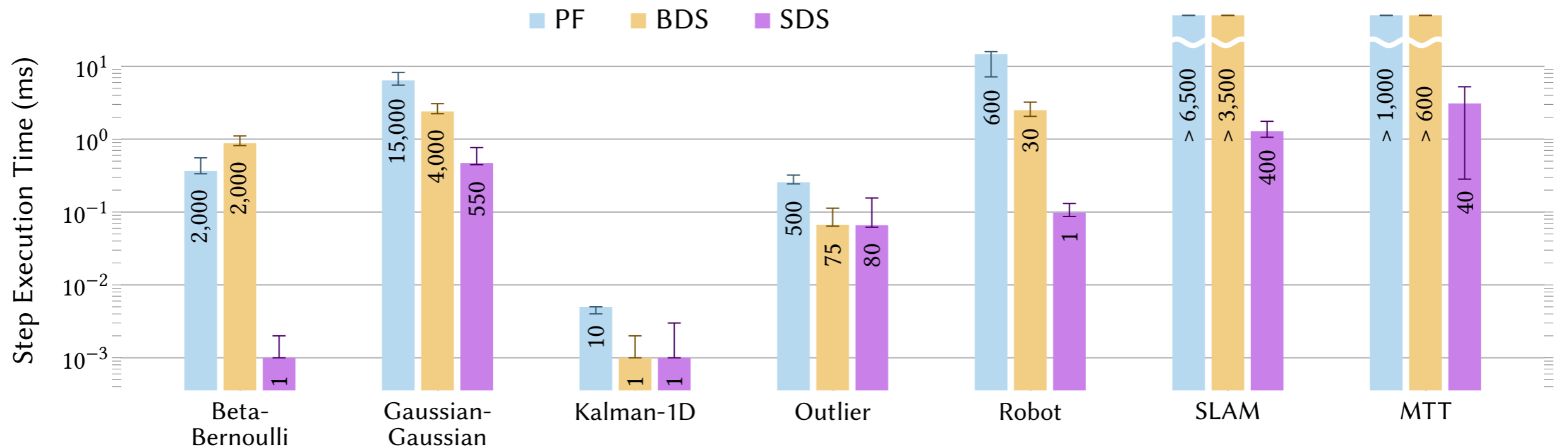


Number of Particles (log scale)

Number of Particles (log scale)

Streaming Delayed Sampling

- Benchmarks illustrate: fixed parameters, trajectory, inference-in-the-loop
- Baseline: accuracy of SDS with 500 particles
- Latency of the inference algorithms to reach comparable accuracy



Conclusion

ProbZelus

- Synchronous language extended with probabilistic constructs
- Inference-in-the-loop
- Efficient streaming inference algorithms

Design, Semantics, Compilation

- Type system to discriminate deterministic and probabilistic processes
- Measure-based co-iterative semantics
- Semantics preserving compilation scheme

Streaming inference

- Adapt particle filtering and delayed sampling to run on stream processors
- Streaming delayed sampling implementation that run in bounded memory